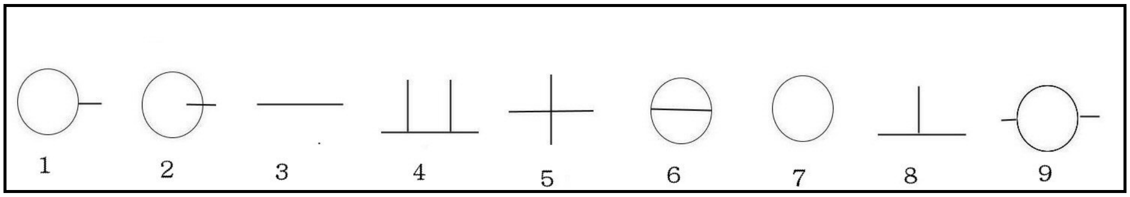
Section 5.3 Some General Ideas of Topology

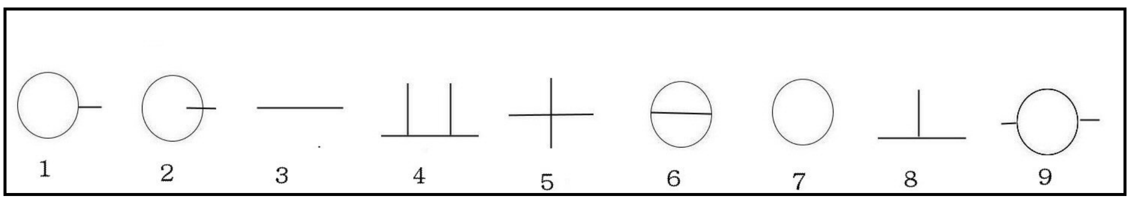
1 **(Letters of the Alphabet)** To a topologist there are 9 letters in the alphabet. Which of the 26 non-topological letters of the alphabet are homeomorphic to the following 9 topological letters.



You can use the 26 non-topological fonts as the following.

A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

Ans:



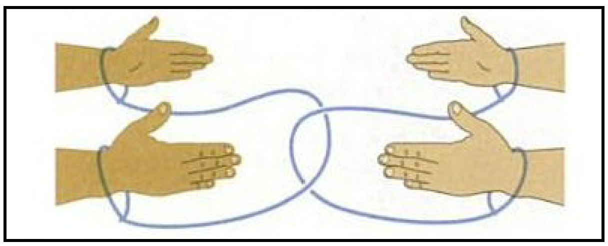
1 = P
2 = Q
3 = C,I,J,L,M,N,S,U,V,W,Z
4 = H,K
1: 5 = X
6 = B
7 = D,O
8 = E,F,G,T,Y
9 = A,R

2. Show the open interval (0,1) is homeomorphic to the real line.
Ans: Draw the proper figure. Student project

3. **(**Möbius **Strip)** Construct a Möbius strip from a strip of paper and answer the following questions by experimentation.

a) The surface of a Möbius strip is not orientable.
b) The surface of a sphere is orientable (draw circles on a basketball).
c) Another topological property of a surface is the number of sides of a surface. Verify that a Möbius strip is not homeomophic to a loop of paper without the half twist at the ends.[1]

4. **(Topological Puzzle)** Mary has just graduated from the police academy and has handcuffed two robbers after robbing a bank. Unfortunately, she has interlocked the chains of the prisoner's handcuffs as illustrated in the following figure. To make matters worse, Mary has lost the keys to the handcuffs. Can you tell Mary how to separate the prisoners without having to cut the chains or cutting off the arms of the prisoners?



Ans: This is a problem that's more fun to solve than read a solution. It is possible.

5. **(Experimenting with the** Möbius **Band)** Take a piece of paper about a foot long and an inch wide. First, bring the ends of the paper together to make a loop but then give one of the ends a half twist and tape the ends together. You now have a band with a half twist in it, called a Möbius band[2]. You are now ready to carry out the following experiments:

a) Take a red pencil and color around the edge of the Möbius band. Continue until you arrive back at the starting point. How many edges are there, the one you colored and the one you . Surprise.

---

[1] Interesting experiments can be carried out the a Möbius strip and its many variants, such as making more than one half-twist, cutting the strips lengthwise down the middle a different number of times.

[2] Möbius bands are not that uncommon in the real world. Often in industrial settings conveyor belts are given a half twist so each side of the belt wears evenly.

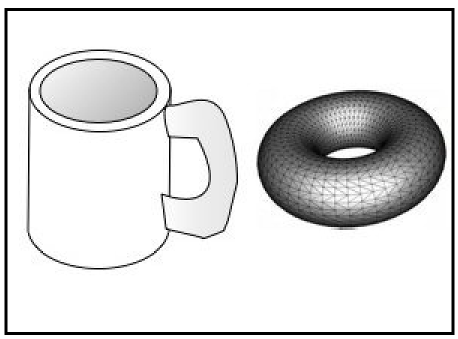
b) Now lets color the sides. Starting at any point draw a line down the middle of the band until you arrive back at the starting point. How many sides are there to the strip? Double surprise.

c) Things just get curiouser and couriouser with the Möbius band. Now take a pair of scissors and cut the band lengthwise down the middle. What do you think will happen?

d) Now create a second Möbius band but this time instead of cutting the band down the middle, cut it so your scissors are about 1/3 the way from one of the edges. The results now are even more surprising.

Ans: It would be unfair to the reader to give away the results.

6. **(The Doughnut and Coffee Cup)** The story goes that one should never give a topologist a cup of coffee else the topologist will begin to chew on the cup thinking that it is a doughnut. While it is true that a doughnut (torus) and coffee cup are homeomorphic images, can you find three other figures in three dimensions that are not homeomoprhic to the doughnut or coffee cup? What are some topological properties of these objects that are different from those of the doughnut or coffee cut?



Ans: A Möbius band is not homeomorphic to a doughnut or coffee cup since it has two sides and the doughnut and coffee cup have only one, and the number of sides is a topological property of a surface. Two other non-homeomorphic figures would be a doughnut with two holes and a doughnut with three holes, all of which are non-homeomorphic since the number of holes in a doughnut is a topological property of a solid figure.

7. **(Euler Characteristic Experiment)** Start with a graph in the plane consisting of only one point and compute the Euler characteristic of the graph. (Remember the regions bounded by edges are faces, not the one unbounded region). Then start adding vertices and edges and keep monitoring the Euler characteristic. It should always be 2.

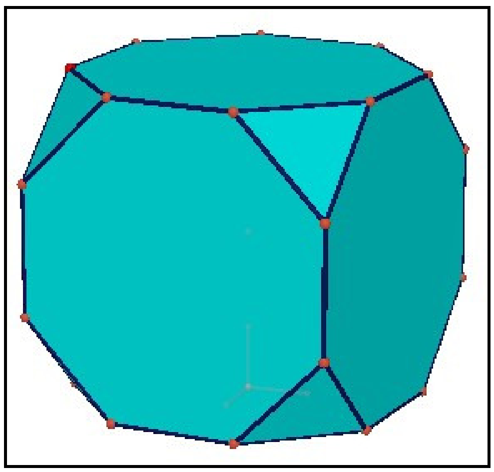
8. **(Euler Characteristic for Planar Polygons)** Show that the Euler characteristic $v - e + f$ of each of the polygons in the plane, square, polygon, hexagon, and octagon is 1.

Ans: We leave this for the reader.

9. **(Euler's Formula)** Carry out the steps of Cauchy's proof of Euler's formula for the tetrahedron and octahedron.

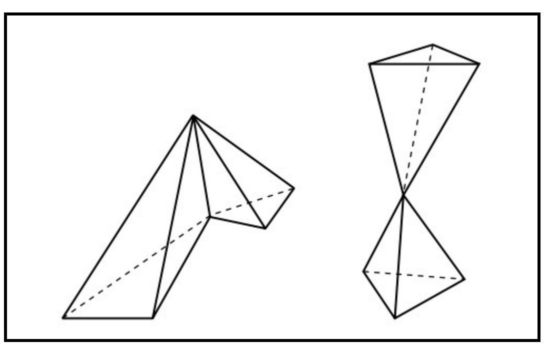
Ans: We leave this fun exercise for the reader.

10**. (Truncated Cube)** A **truncated solid** is a polyhedra with its vertices clipped off. Find the number of vertices, edges, and faces of the truncated cube and show that it has an Euler characteristic of 2.



Ans: The truncated cube has $v = 24,\ e = 36,\ f = 14$ for an Euler characteristic of 2.

11. Non-convex Polyhedra Show that the following 2 non-convex polyhedra each have and Euler Characteristic of 3.



;

ΦΙΔΞΠΝΘ